

# Intensity-dependent parameterization of elevation effects in precipitation analysis

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**Abstract.** Elevation effects in long-term (monthly to inter-annual) precipitation data have been widely studied and are taken into account in the regionalization of point-like precipitation amounts by using methods like external drift kriging and cokriging. On the daily or hourly time scale, precipitation-elevation gradients are more variable, and difficult to parameterize. For example, application of the annual relative precipitation-elevation gradient to each 12-h sub-period reproduces the annual total, but at the cost of a large root-mean-square error. If the precipitation-elevation gradient is parameterized as a function of precipitation rate, the error can be substantially reduced. It is shown that the form of the parameterization suggested by the observations conforms to what one would expect based on the physics of the orographic precipitation process (the seeder-feeder mechanism). At low precipitation rates, orographic precipitation is “conversion-limited”, thus increasing roughly linearly with precipitation rate. At higher rates, orographic precipitation becomes “condensation-limited” thus leading to an additive rather than multiplicative orographic precipitation enhancement. Also it is found that for large elevation differences it becomes increasingly important to take into account those events where the mountain station receives precipitation but the valley station remains dry.

## 1 Introduction

In mountainous terrain, elevation differences strongly contribute to the small-scale spatial variability of precipitation. The effect is most pronounced for long accumulation periods such as monthly, annual, or inter-annual. Due to partial cancellation of non-orographic spatial patterns on these

timescales, elevation becomes the dominant factor determining the small-scale structure of the precipitation field. External drift kriging and cokriging methods are employed to derive statistical relationships between precipitation data and the terrain and to interpolate the precipitation field between point observations (Daly et al., 1994; Goovaerts, 2000; Guan et al., 2005; Hunter and Meentemeyer, 2005).

However, a long-term precipitation distribution is the sum of individual short-term distributions, and elevation effects must be present, even if not always visually discernible, in arbitrarily short precipitation analyses as well. In Austria, the recent development of flood forecasting models for small Alpine catchments has created a need for high-resolution (1 km), short-term (15 min) precipitation analyses. While there have been many studies on long-term elevation-precipitation relationships (Smith, 1979; Barry, 1992; Baisist et al., 1994; Daly et al., 1994; Kiefer Weisse and Bois, 2001), little is known about such relationships at shorter time scales. Differences in meteorological conditions (static stability, flow direction and strength, freezing level) between individual precipitation events lead to large variations of the precipitation-elevation relationship. Moreover, when short time periods are considered, precipitation will frequently occur only in parts of an area, which makes the application of kriging methods difficult.

As part of a project in which a flood prediction system for the Austrian federal province of Salzburg is being developed, elevation effects on precipitation over 12-h time intervals are investigated. An objective of the study was to determine the most important factors affecting the strength of the precipitation-elevation relationship. It turned out that correlations with quantities like wind speed, stability, or temperature (as an indirect measure of freezing level) were generally too weak to be used in a parameterization. Only precipitation amount itself exhibited a correlation with the elevation effect sufficiently robust to serve as a basis for a parameterization. Another objective was to answer the question: If



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**Table 1.** Topographic characteristics of station pairs used in the analysis. The last column gives the direction of the valley station relative to the mountain station.

#	Station	z(m)	$\Delta z$ (m)	$\Delta x$ (m)	Dir
1	Hahnenkamm	1790	1046	3800	NNE
	Kitzbüchel	744			
2	Loferer Alm	1623	998	4200	ESE
	Lofer	625			
3	Schmittenhöhe	1973	1207	4400	E
	Zell am See	766			
4	Feuerkogel	1618	1191	4300	E
	Gmunden	427			
5	Rax	1547	1061	4900	ESE
	Reichenau	486			

a long-term (annual or inter-annual) precipitation distribution for a mountainous area is given, is it possible to derive a parameterization for short time intervals which generates analyses that reproduce the long-term distribution when accumulated over the long-term period?

The paper is organized as follows. Section 2 describes the study area and data set used in the study. Section 3 presents a parameterization which allows computation of the elevation effect for different 12-h rainfall amounts from the long-term precipitation-elevation gradient. Results are discussed in Sect. 4 which also illustrates the application of the parameterization in the operational INCA (Integrated Nowcasting through Comprehensive Analysis) system.

## 2 Data and study area

In the study of elevation effects on precipitation it is necessary to define the spatial scale on which a relationship is supposed to be valid. In the Alpine area no systematic increase of precipitation with elevation exists on the 50–100 km scale. This is not meant to imply that station pairs do not show a significant correlation over this distance (and even beyond, as shown for daily values by Ahrens, 2006). It is meant in the sense that if topography and precipitation are smoothed over such a scale they do not exhibit a simple relationship. Precipitation increases from the Alpine foreland towards the northern and southern upslope areas and generally decreases towards the interior Alpine areas, in spite of higher terrain there (Frei and Schär, 1998). These areas experience precipitation shielding due to mountain-range blocking and upslope effects. This is a familiar pattern that can be found in many other areas such as the Pacific coastal mountains of the US and Canada, or the mountains of Norway and Sweden. However, superimposed on this larger scale are patterns due to

individual mountain ridges and valleys (5–10 km scale). It is the variation on this horizontal scale we attempt to parameterize. It also appears to be the optimal scale for the application of elevation-precipitation relationships (Daly et al., 1994; Sharples et al., 2005). Moreover, the average distance between real-time rain gauge stations in the Austrian Alps is  $\sim 20$  km which means they already capture most of the meso- $\beta$  scale precipitation variations. This is another reason why our study focuses on the rather local (5–10 km) meso- $\gamma$  scale increase of precipitation from a valley floor to the surrounding ridges and peaks.

Table 1 lists the station pairs used in the analysis. The horizontal distance between mountain and valley stations is about 4 km, the vertical distance is about 1 km. On the meso- $\beta$  scale, station pairs 2 and 4 are located in the primary northern Alpine upslope precipitation belt, whereas station pairs 1, 3, and 5 are experiencing already some downstream sheltering. Pairs 1–4 are located well north of the main Alpine crest, pair 5 is situated at the eastern end of the Alpine chain. For this study we used 12-h precipitation observations (06:00–18:00 UTC, 18:00–06:00 UTC) from the 11-yr period 1995–2005. The observations were corrected for wind effects following the method of Skoda and Filipovic (2007) which estimates a correction factor as a function of precipitation intensity, wind speed, and wet-bulb temperature (for the distinction between snow and rain).

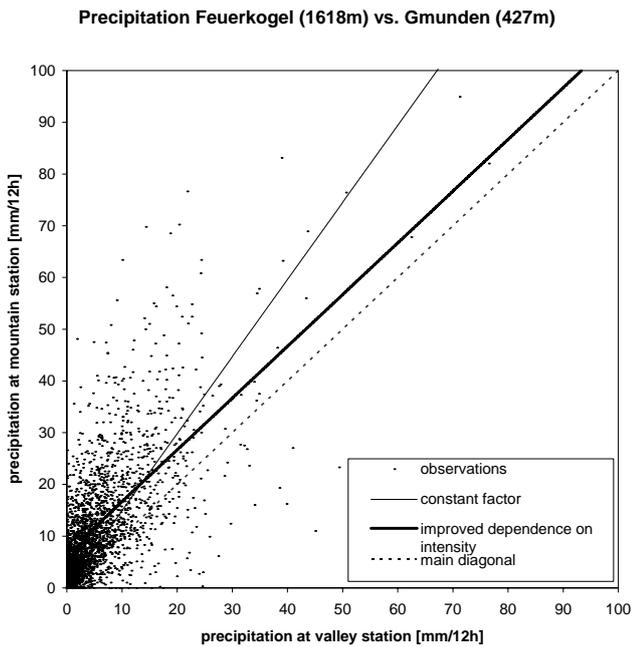
## 3 Parameterization

Figure 1 shows 12-h precipitation amounts for station pair 4, which has been chosen because it is located in the most upstream and most textbook-like primary upslope precipitation area. Even so, the scatter is large. If anything, the Figure suggests a weak linear relationship with a slope of 2–3 (not shown) for valley precipitation amounts up to 20–25 mm. Applying such a factor to all precipitation events, however, would lead to a strong overestimation of annual precipitation at the mountain station. If a linear relationship would be used, the slope that best reproduces the annual total at the mountain station is close to 1.5 (thin continuous line). By doing so we would underestimate the elevation effect for the large number of events with less than 10 mm at the valley station.

As one step beyond a linear relationship we propose the following parameterization of the dependence of mountain precipitation  $P_{\text{mtn}}$  on valley precipitation  $P_{\text{val}}$

$$P_{\text{mtn}} = \begin{cases} P_{\text{val}}(a - bP_{\text{val}}) & P_{\text{val}} \leq P_c \\ P_{\text{val}} + (a - 1 - bP_c)P_c & P_{\text{val}} \geq P_c \end{cases}, \quad (1)$$

where  $P_c = (a - 1) / (2b)$ . At  $P_{\text{val}} = P_c$  the relationship changes from parabolic to linear. The location of  $P_c$ , i.e. its dependence on  $a$  and  $b$ , follows from the condition  $dP_{\text{mtn}}/dP_{\text{val}} = 1$



**Fig. 1.** Precipitation at mountain station vs. precipitation at valley station for individual 12-h amounts for station pair 4. The bold curve shows the parameterized relationship Eqs. (1), (4), (5). For comparison, the thin line shows the simple linear relationship Eq. (3).

which ensures continuity of slope at  $P_{val}=P_c$ . Equation (1) can also be written

$$P_{mtn} = \begin{cases} P_{val} \left[ 1 + (a - 1) \left( 1 - \frac{P_{val}}{2P_c} \right) \right] & P_{val} \leq P_c \\ P_{val} + \frac{a-1}{2} P_c & P_{val} \geq P_c \end{cases}, \quad (2)$$

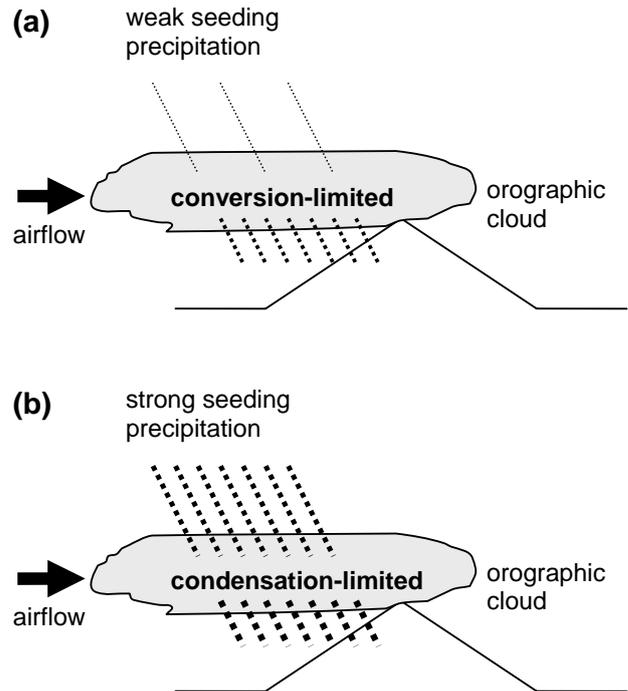
where  $b$  has been expressed in terms of  $a$  and  $P_c$ . The parameter  $a$  is the ratio between mountain and valley precipitation in the limit of small valley precipitation. The parameter  $b$  is a measure of how strongly the ratio between mountain and valley precipitation decreases with increasing valley precipitation. For small values of valley precipitation, Eq. (1) reduces to the simple linear relationship

$$P_{mtn} = a P_{val}. \quad (3)$$

As  $P_{val}$  increases, but remains below the critical value  $P_c$ , the ratio  $P_{mtn}/P_{val}$  decreases, and mountain precipitation as given by Eq. (1) becomes a parabolic function of valley precipitation. Above the critical value, the relationship between  $P_{val}$  and  $P_{mtn}$  is additive. For a given value of the parameter  $a$ , an optimum value of  $b$  is computed from a given ratio of long-term (inter-annual) precipitation totals at the mountain and valley stations

$$A = \frac{(P_{mtn})_{ann}}{(P_{val})_{ann}} \quad (4)$$

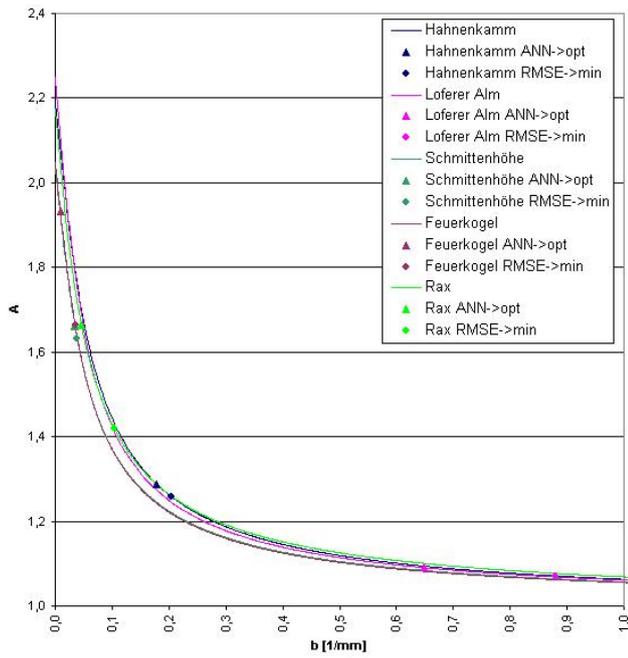
by minimizing the mountain precipitation root-mean-square error (RMSE) when predicted by valley precipitation. Note



**Fig. 2.** If the non-orographic (seeding) precipitation is weak (a), orographic enhancement is limited by conversion. If seeding is strong (b), orographic enhancement is limited by condensation.

that the numbers given below refer to values of  $A$ ,  $a$ ,  $b$  normalized to an elevation difference of 1000 m between mountain and valley station. The different elevation differences between station pairs listed in Table 1 have been taken into account by normalizing the inter-annual precipitation ratio  $A$  in Eq. (4) by  $A \cdot 1000/\Delta z$ . Similarly, application of the parameterization to an arbitrary height difference  $\Delta z$  means replacing  $P_{mtn}/P_{val}$  as obtained from Eq. (1) by  $(P_{mtn}/P_{val})\Delta z/1000$ .

The different behaviour of orographic precipitation enhancement in the limit of small and high precipitation rates implied by Eq. (1) is consistent with the physics of the seeder-feeder process (Smith, 1979; Cotton and Anthes, 1989). If the non-orographic (seeding) precipitation is weak (Fig. 2a), orographic enhancement is limited by conversion. Only a small fraction of the condensate produced in the orographic cloud is washed out. Increasing the seeding therefore leads to a roughly proportional increase of precipitation at the ground. If the seeding rate is high (Fig. 2b), washout of condensate is very efficient, and orographic enhancement becomes limited by condensation. An increase in the intensity of seeding does not lead to a proportional increase of precipitation at the ground. The orographic effect is basically additive in such a case. As shown analytically by Haiden (1995) the critical seeding rate, above which the process becomes limited by condensation rather than accretion efficiency, increases with wind speed.



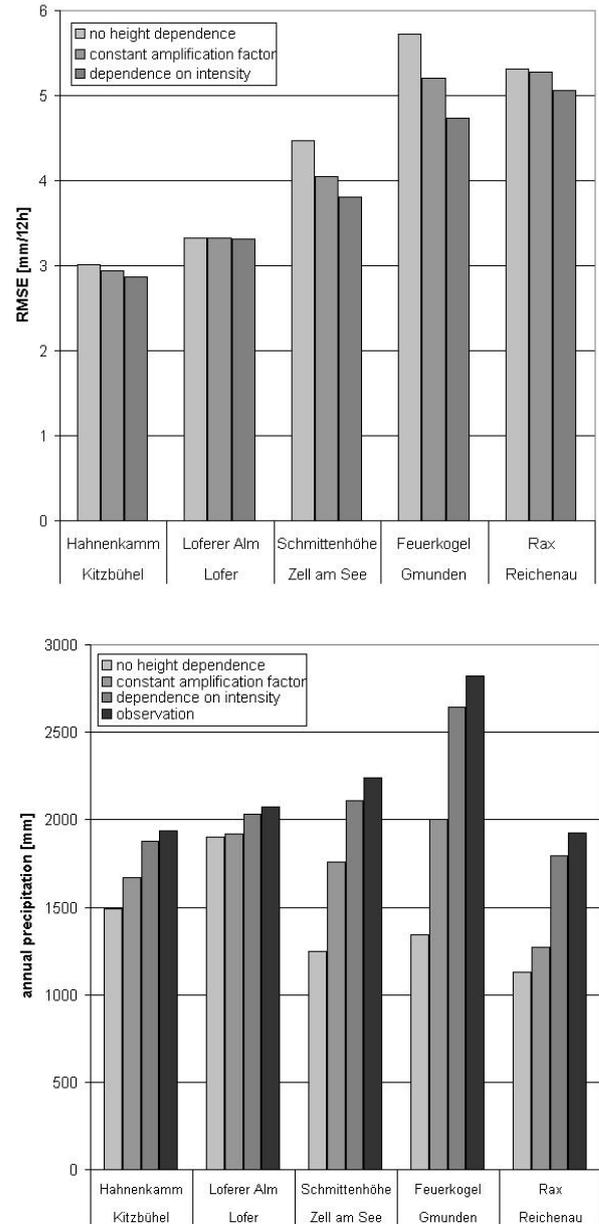
**Fig. 3.** Relationship Eq. (5) between the inter-annual mountain/valley precipitation ratio  $A$  and the coefficient  $b$  for the 5 different locations. Diamonds indicate points of minimal 12-h RMSE, triangles indicate points of best reproduction of the annual total.

**4 Results and discussion**

In a first step, both  $a$  and  $b$  were varied independently for all station pairs. Interestingly it was found that the optimal value of  $a$  (as measured by mountain precipitation RMSE), representing the precipitation enhancement for small precipitation amounts, differed little between pairs and could be set to the location-independent value of 2.2 without significantly increasing the RMSE. Thus we applied Eq. (1) with  $a=2.2$  to the 11-yr dataset, varying the coefficient  $b$ , thereby obtaining different inter-annual ratios  $A$ . The resulting relationship between  $A$  and  $b$  is quite similar for all 5 station pairs (Fig. 3), confirming the viability of the approach. The similarity appears to be a result of the broadly similar precipitation climate at the selected locations. The relationship can be analytically fitted by

$$b(A) = \frac{1}{c_1(A - 1)} - \frac{1}{c_2}, \tag{5}$$

where  $c_1=16.0$  mm and  $c_2=18.6$  mm (12-h totals). Different values of  $b$  are found for a given  $A$  when instead of minimizing the 12-h RMSE, the condition of reproducing the long-term totals is prescribed (Fig. 3). Figure 4 shows the RMSE values and annual totals for using no elevation correction, using the simplified version Eq. (3), and using the full parameterization Eq. (1). The height correction generally gives an improvement compared to using none. The parameterization reduces the RMSE somewhat more than the simplified



**Fig. 4.** Mountain station precipitation RMSE (top) and annual precipitation totals (bottom) for the cases of no elevation correction, for using simplified version Eq. (3), and for using the parameterization Eq. (1). Observed annual totals shown in black.

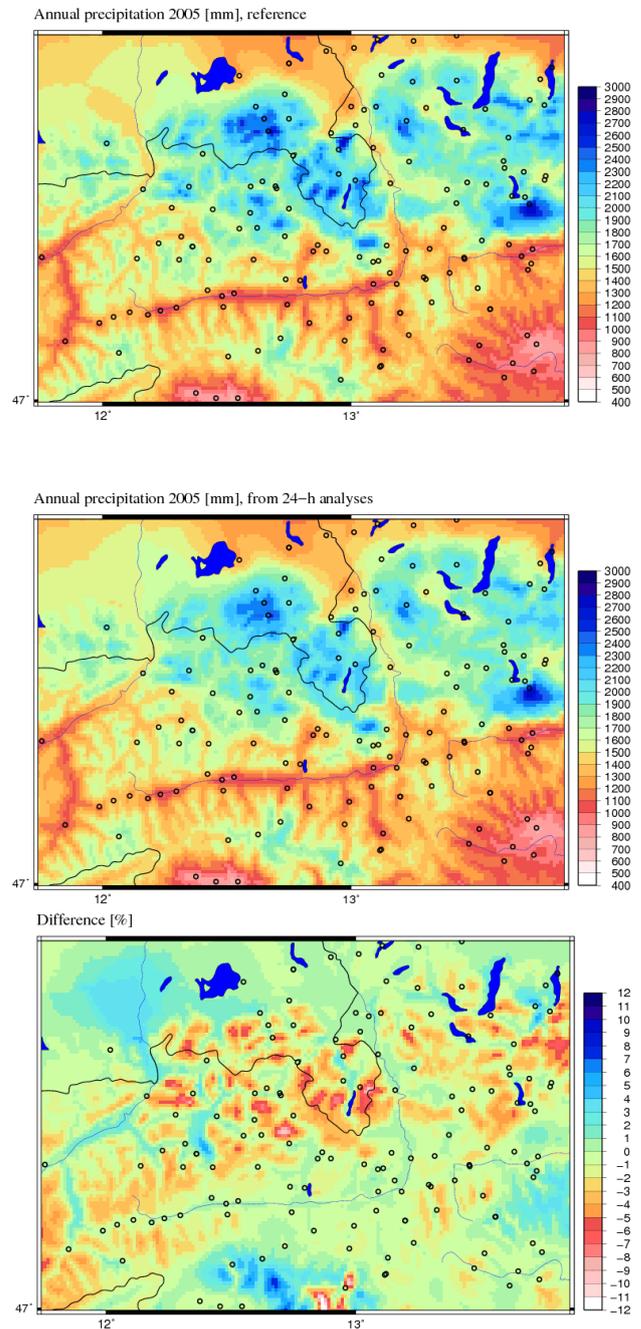
version. Both error reductions are modest, amounting to only about 5–10% of the RMSE. More importantly, however, the parameterization gives annual totals that are much closer to the observed ones. Thus it is a useful method for distributing a given long-term average precipitation enhancement to individual 12-h intervals.

The parameterization does not cover events where the mountain station receives precipitation but the valley station remains dry. This situation becomes increasingly likely for

larger differences of altitude between the two stations, for drier layers of air in-between, and for weaker precipitation events. To further study this problem, we performed the analysis on an independent set of station pairs with vertical distances that deviate from the previous value of approximately 1000 m. These are Schöckl (1445 m) and Sankt Radegund (725 m), Villacher Alpe (2164 m) and Villach (494 m), and Sonnblick (3105 m) and Kolm-Saigurn (1618 m). Note that the parameters  $a$  and  $b$  as well as the inter-annual precipitation ratio  $A$  are again normalized to a vertical distance of 1000 m in all cases.

It was mentioned above that the coefficient  $a$ , which represents the elevation dependence for small precipitation amounts at the valley station, was found to exhibit smaller variations than  $b$  and was therefore set to a constant value. This allowed the derivation of a parameterization based on a one-to-one relationship between the inter-annual ratio  $A$  and the parameter  $b$ . The specific value of  $a=2.2$  was chosen to best satisfy the data of the five station pairs in Table 1. However, the extension of the analysis to station pairs with more widely varying vertical distances suggests significant variations of  $a$  as well. For station pairs with large vertical distances, i.e. the Villacher Alpe and Sonnblick data sets, the optimum values for  $a$  that minimize the mountain precipitation RMSE are 6.4 and 30.4, respectively. For  $b$ , these values are 3.7 and 50.1  $\text{mm}^{-1}$ , implying a very large relative orographic enhancement for weak precipitation events and an early transition to the additive part of relationship Eq. (1) as events get stronger. This suggests that cases with mountain-only precipitation become increasingly dominant as the vertical distance between the stations increases.

The above parameterization of elevation dependence is operationally used in the INCA system (Haiden et al., 2009), which provides the meteorological input for flood prediction models in Austria (Komma et al., 2007). The system generates real-time 15-min and 24-h precipitation analyses based on radar and surface station data. For application of the parameterization to durations  $D$  other than 12 h, precipitation values  $P_D$  are normalized to their equivalent 12-h amounts  $P_{12}$  using square-root temporal scaling  $P_{12}=P_D\sqrt{12/D}$ . Figure 5 shows the annual precipitation distribution of the year 2005 for the area of Salzburg, computed by INCA in two different ways. No radar data has been used in order to simplify interpretation of the results. First, annual precipitation totals observed at the stations were spatially interpolated using the annual average elevation factor  $A$  derived from a climatological precipitation distribution of the area. Second, 24-h precipitation amounts observed at the stations were spatially interpolated using Eqs. (1) and (5), and then accumulated over the year 2005. The comparison shows that the summation of the 24-h analyses reproduces the annual precipitation distribution to within 5% in most areas. The largest underestimations (up to about 10%) occur at mountain tops and ridges located in the primary northern upslope belt.



**Fig. 5.** Areal precipitation distribution of the year 2005 in the Salzburg area obtained by interpolating observed annual totals with a climatologically derived elevation dependence (top), obtained by accumulation of 24-h analyses with elevation dependence parameterized according to Eq. (1) (center), and the difference between the two fields (bottom). Circles show station locations, black line indicates the Austrian border.

## 5 Conclusions

In order to improve high-resolution precipitation analyses over mountainous terrain for short time scales (24 h and smaller), a method of deriving short-term elevation gradients from long-term enhancement ratios is presented. The analytical form of the parameterization is motivated by the physics of the seeder-feeder mechanism. For small precipitation amounts it is multiplicative, for larger amounts additive. The coefficients of the parameterization appear to be fairly robust for different station pairs as long as their elevation difference is similar ( $\sim 1$  km). For larger elevation differences (1.5 km and higher), the effect of mountain-only precipitation events becomes increasingly important. Taking this into account in the framework of the current parameterization would imply very large enhancement factors for small precipitation amounts, and a rapid transition to additive enhancement. Due to the limited number of station pairs which represent such large elevation differences (just one), such an extension was not attempted. For the operational application of the elevation dependence in the INCA analysis and nowcasting system the conservative estimate derived from the station pairs in Table 1 is used. It leads to an underestimation of mountain precipitation on the order of 5–10%, which is an acceptable error considering the fundamental difficulty posed by mountain-only precipitation events on unobserved peaks and ridges, which cannot be solved by means of parameterization.

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## References

- Ahrens, B.: Distance in spatial interpolation of daily rain gauge data, *Hydrol. Earth Syst. Sci.*, 10, 197–208, 2006, <http://www.hydrol-earth-syst-sci.net/10/197/2006/>.
- Barry, R. G.: *Mountain Weather and Climate*, Routledge, 313 pp., 1992.
- Basist, A., Bell, G. D., and Meentemeyer, V.: Statistical relationships between topography and precipitation, *J. Climate*, 7, 1305–1315, 1994.
- Cotton, W. R. and Anthes, R. A.: *Storm and Cloud Dynamics*, Academic Press, San Diego, 880 pp., 1989.
- Daly, C., Neilson, R. P., and Phillips, D. J.: A statistical-topographic model for mapping climatological precipitation over mountainous terrain, *J. Appl. Meteor.*, 33, 140–158, 1994.
- Frei, C. and Schär, C.: A precipitation climatology of the Alps from high-resolution rain-gauge observations, *Int. J. Climatol.*, 18, 873–900, 1998.
- Goovaerts, P.: Geostatistical approaches for incorporating elevation into the spatial interpolation of rainfall, *J. Hydrol.*, 228, 113–129, 2000.
- Guan, H., Wilson, J. L., and Makhnin, O.: Geostatistical mapping of mountain precipitation incorporating autosearched effects of terrain and climatic characteristics, *J. Hydromet.*, 6, 1018–1031, 2005.
- Haiden, T.: Analytical solution of washout in stratiform clouds. *Contrib. Atmos. Phys.*, 68, 3–14, 1995.
- Haiden, T., Kann, A., Pistotnik, G., Stadlbacher, K., and Wittmann, C.: Integrated Nowcasting through Comprehensive Analysis (INCA) – System description, ZAMG report, p. 60, [http://www.zamg.ac.at/fix/INCA\\_system.pdf](http://www.zamg.ac.at/fix/INCA_system.pdf), 2009.
- Hunter, R. D. and Meentemeyer, R. K.: Climatologically aided mapping of daily precipitation and temperature, *J. Appl. Meteor.*, 44, 1501–1510, 2005.
- Kiefer Weisse, A. and Bois, P.: Topographic effects on statistical characteristics of heavy rainfall and mapping in the French Alps, *J. Appl. Meteor.*, 40, 720–740, 2001.
- Komma, J., Reszler, C., Blöschl, G., and Haiden, T.: Ensemble prediction of floods - catchment non-linearity and forecast probabilities, *Nat. Hazards Earth Syst. Sci.*, 7, 431–444, 2007, <http://www.nat-hazards-earth-syst-sci.net/7/431/2007/>.
- Sharples, J. J., Hutchinson, M. F., and Jellett, D. R.: On the horizontal scale of elevation dependence of Australian monthly precipitation, *J. Appl. Meteor.*, 44, 1850–1865, 2005.
- Skoda, G. and Filipovic, N.: Multi-data precipitation analysis. Austrian Academy of Sciences, Final report, Vienna, 17 pp., 2007.
- Smith, R. B.: The influence of mountains on the atmosphere, *Adv. Geophys.*, 21, Academic Press, 87–230, 1979.